Bishop Hooper CE Primary School,

Mathematics Calculation Policy

Overview of calculation strategies - See timetable for suggested introduction (Appendix A)

Early Years into KS1

Practical, oral and mental activities to understand calculation.

Personal methods of recording. [use of (+ , - , =) encouraged where appropriate to child's development]

Key Stage 1

Methods of recording / jottings to support calculation (e.g. partitioning)

Introduce signs and symbols ***(+ / - in Year 1 and x / ÷ in Year 2)***

Use images such as empty number lines to support mental and informal calculation.

An opportunity is given to the more able Yr2 pupils to experience more efficient informal methods. [see Yr3]

Year 3

More efficient informal written methods / jottings – expanded methods and efficient use of number lines.

Years 4-6

Continue using efficient informal methods (expanded addition and subtraction, grid multiplication, division by chunking) and number lines. Develop these to larger numbers and decimals where appropriate.

Begin to develop efficient written methods (standard / compact methods) for all four operations

When faced with a calculation assessment, children are able to decide which method is most appropriate and have strategies to check its accuracy.

> ***Whatever method is chosen (in any year group), it must still be underpinned by a secure and appropriate knowledge of number facts***

By the end of Year 6, children should:

- have a secure knowledge of number facts and a good understanding of the four operations in order to:
    - carry out calculations mentally when using one-digit and two-digit numbers
    - use particular strategies with larger numbers when appropriate
- use notes and jottings to record steps and part answers when using longer mental methods
- **have an efficient, reliable, compact written method of calculation for each operation that children can apply with confidence when undertaking calculations that they cannot carry out mentally;**

Children should always **look at the actual numbers (not the size of the numbers**) before attempting any calculation to determine whether or not they need to use a written method.

Therefore, the key question that children should always ask themselves before attempting a calculation is: -

Can I do it in my head?

Mental methods of calculation

Oral and mental work in mathematics is essential, particularly so in calculation. Early practical, oral and mental work must lay the foundations by providing children with a good understanding of how the four operations build on efficient counting strategies and a secure knowledge of place value and number facts. Later work must ensure that children recognise how the operations relate to one another and how the rules and laws of arithmetic are to be used and applied. Ongoing oral and mental work provides practice and consolidation of these ideas. It must give children the opportunity to apply what they have learned to particular cases, exemplifying how the rules and laws work, and to general cases where children make decisions and choices for themselves.

The ability to calculate mentally forms the basis of all methods of calculation and has to be maintained and refined. A good knowledge of numbers or a ‘feel’ for numbers is the product of structured practice and repetition. It requires an understanding of number patterns and relationships developed through directed enquiry, use of models and images and the application of acquired number knowledge and skills. Secure mental calculation requires the ability to:

- recall key number facts instantly – for example, all addition and subtraction facts for each number to at least 20 **(Year 2),** sums and differences of multiples of 10 **(Year 3)**
- Recall multiplication facts; Yr2 -2 , 5 & 10, Yr3- 3 , 4**,** 8 and multiplication facts up to 12 × 12 (Year 4);
- use taught strategies to work out the calculation – for example, recognise that addition can be done in any order and use this to add mentally a one-digit number or a multiple of 10 to a one-digit or two-digit number (Year 1), partition two-digit numbers in different ways including into multiples of ten and one and add the tens and ones separately and then recombine (Year 2), when applying mental methods in special cases (Year 5);
- understand how the rules and laws of arithmetic are used and applied – for example, to add or subtract mentally combinations of one-digit and two-digit numbers (Year 3), and to calculate mentally with whole numbers and decimals (Year 6).

Objectives

The objectives in the New Curriculum 2014 show the progression in children's use of written methods of calculation in the strands 'Number – addition and subtraction', 'Number – multiplication and division' and 'Number – fractions'

Number EYFS	
Foundation • using quantities and objects they add and subtract single digit numbers and count on or back to find the answers. • they solve problems including doubling, halving and sharing in a practical situation. • counting on and back in 1s, 2, 5 and 10.	

Number – addition and subtraction; multiplication and division; fractions Y1-3	Number– addition and subtraction; multiplication and division; fractions Y4-6
Year 1 • read, write and interpret mathematical statements involving addition (+), subtraction (−) and equals (=) signs • represent and use number bonds and related subtraction facts within 20 • add and subtract one-digit and two-digit numbers to 20, including 0 • solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as 7 = ? – 9 • solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher • recognise, find and name a half as 1 of 2 equal parts of an object, shape or quantity • recognise, find and name a quarter as 1 of 4 equal parts of an object, shape or quantity	**Year 4** • add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate • estimate and use inverse operations to check answers to a calculation • solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why • recall multiplication and division facts for multiplication tables up to 12 × 12 • use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together 3 numbers • recognise and use factor pairs and commutativity in mental calculations • multiply two-digit and three-digit numbers by a one-digit number using formal written layout • solve problems involving multiplying and adding, including using the distributive law to multiply two-digit numbers by 1 digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects • recognise and show, using diagrams, families of common equivalent fractions • count up and down in hundredths; recognise that hundredths arise when dividing an object by 100 and dividing tenths by 10 • solve problems involving increasingly harder fractions to calculate quantities, and fractions to divide quantities, including non-unit fractions where the answer is a whole number • add and subtract fractions with the same denominator • recognise and write decimal equivalents of any number of tenths or hundreds • recognise and write decimal equivalents to $\frac{1}{4}, \frac{1}{2}, \frac{3}{4}$ • find the effect of dividing a one- or two-digit number by 10 and 100, identifying the value of the digits in the answer as ones, tenths and hundredths • round decimals with 1 decimal place to the nearest whole number • compare numbers with the same number of decimal places up to 2 decimal places • solve simple measure and money problems involving fractions and decimals to 2 decimal places

Year 2

- solve problems with addition and subtraction:
  - using concrete objects and pictorial representations, including those involving numbers, quantities and measures
  - applying their increasing knowledge of mental and written methods
- recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100
- add and subtract numbers using concrete objects, pictorial representations, and mentally, including:
  - a two-digit number and 1s
  - a two-digit number and 10s
  - 2 two-digit numbers
  - adding 3 one-digit numbers
- show that addition of 2 numbers can be done in any order (commutative) and subtraction of 1 number from another cannot
- recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems
- recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers
- calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (×), division (÷) and equals (=) signs
- show that multiplication of 2 numbers can be done in any order (commutative) and division of 1 number by another cannot
- solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts
- recognise, find, name and write fractions $\frac{1}{3}$, $\frac{1}{4}$, $\frac{2}{4}$ and $\frac{3}{4}$ of a length, shape, set of objects or quantity
- write simple fractions, for example $\frac{1}{2}$ of 6 = 3 and recognise the equivalence of $\frac{2}{4}$ and $\frac{1}{2}$

Year 5

- add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction)
- add and subtract numbers mentally with increasingly large numbers
- use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy
- solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why
- identify multiples and factors, including finding all factor pairs of a number, and common factors of 2 numbers
- know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers
- establish whether a number up to 100 is prime and recall prime numbers up to 19
- multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers
- multiply and divide numbers mentally, drawing upon known facts
- divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context
- multiply and divide whole numbers and those involving decimals by 10, 100 and 1,000
- recognise and use square numbers and cube numbers, and the notation for squared (^{2}) and cubed (^{3})
- solve problems involving multiplication and division, including using their knowledge of factors and multiples, squares and cubes
- solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign
- solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates
- compare and order fractions whose denominators are all multiples of the same number
- identify, name and write equivalent fractions of a given fraction, represented visually, including tenths and hundredths
- recognise mixed numbers and improper fractions and convert from one form to the other and write mathematical statements > 1 as a mixed number [for example, $\frac{2}{5} + \frac{4}{5} = \frac{6}{5} = 1\frac{1}{5}$]
- add and subtract fractions with the same denominator, and denominators that are multiples of the same number
- multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams
- read and write decimal numbers as fractions [for example, 0.71 = $\frac{71}{100}$]
- recognise and use thousandths and relate them to tenths, hundredths and decimal equivalents
- round decimals with 2 decimal places to the nearest whole number and to 1 decimal place
- read, write, order and compare numbers with up to 3 decimal places
- solve problems involving number up to 3 decimal places
- recognise the per cent symbol (%) and understand that per cent relates to ‘number of parts per 100’, and write percentages as a fraction with denominator 100, and as a decimal fraction
- solve problems which require knowing percentage and

	decimal equivalents of $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{2}{5}$, $\frac{4}{5}$ and those fractions with a denominator of a multiple of 10 or 25

Year 3	Year 6
• count from 0 in multiples of 4, 8, 50 and 100; find 10 or 100 more or less than a given number • recognise the place value of each digit in a 3-digit number (100s, 10s, 1s) • compare and order numbers up to 1,000 • identify, represent and estimate numbers using different representations • read and write numbers up to 1,000 in numerals and in words • solve number problems and practical problems involving these ideas • recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables • write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods • solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects • count up and down in tenths; recognise that tenths arise from dividing an object into 10 equal parts and in dividing one-digit numbers or quantities by 10 • recognise, find and write fractions of a discrete set of objects: unit fractions and non-unit fractions with small denominators • recognise and use fractions as numbers: unit fractions and non-unit fractions with small denominators • recognise and show, using diagrams, equivalent fractions with small denominators • add and subtract fractions with the same denominator within one whole [for example, $\frac{5}{7}+\frac{1}{7}=\frac{6}{7}$] • compare and order unit fractions, and fractions with the same denominators • solve problems that involve all of the above	• multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication • divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context • divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context • perform mental calculations, including with mixed operations and large numbers • identify common factors, common multiples and prime numbers • use their knowledge of the order of operations to carry out calculations involving the 4 operations • solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why • solve problems involving addition, subtraction, multiplication and division • use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy • use common factors to simplify fractions; use common multiples to express fractions in the same denomination • compare and order fractions, including fractions >1 • add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions • multiply simple pairs of proper fractions, writing the answer in its simplest form [for example, $\frac{1}{4}\times\frac{1}{2}=\frac{1}{8}$] • divide proper fractions by whole numbers [for example, $\frac{1}{3}\div 2=\frac{1}{6}$] • associate a fraction with division and calculate decimal fraction equivalents [for example, 0.375] for a simple fraction [for example, $\frac{3}{8}$] • identify the value of each digit in numbers given to 3 decimal places and multiply and divide numbers by 10, 100 and 1,000 giving answers up to 3 decimal places • multiply one-digit numbers with up to 2 decimal places by whole numbers • use written division methods in cases where the answer has up to 2 decimal places • solve problems which require answers to be rounded to specified degrees of accuracy • recall and use equivalences between simple fractions, decimals and percentages, including in different contexts

Appendix A
Written methods for addition of whole numbers

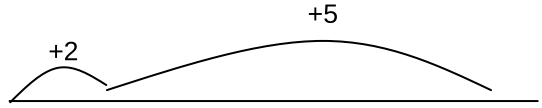
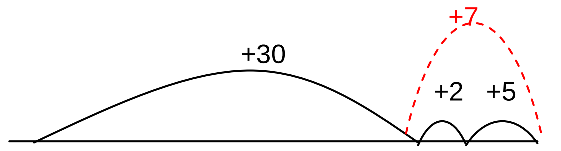
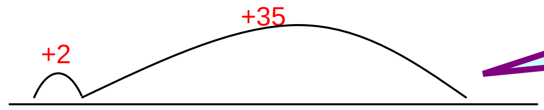
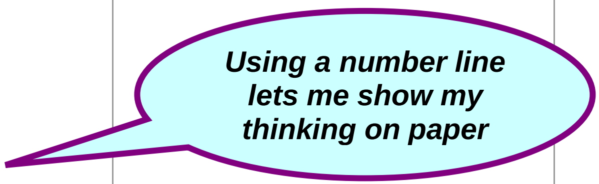
The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence.

Children need to acquire **one efficient written method of calculation for** addition which they know they can rely on **when mental methods are not appropriate.**

To add successfully, children need to be able to:

- **recall all addition pairs to 9 + 9 and complements in 10;**
- **add mentally a series of one-digit numbers, such as 5 + 8 + 4;**
- **add multiples of 10 (such as 60 + 70) or of 100 (such as 600 + 700) using the related addition fact, 6 + 7, and their knowledge of place value;**
- **partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways.**

Note: It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for addition.

Year group	Main method	Alternative method(s)
Year R	Using practical methods with a wide range of apparatus to solve problems. Use a number line and 100 square where appropriate.	
Year 1	Continue to use a number line and 100 square in order to complete addition by "counting on."	
	Stage 1: The empty number line	**Partition one of the numbers**
Year 2 / 3 (Add speech bubbles at start of section – using '**This is the way we do it"**	Steps in addition can be recorded on a number line. The steps often bridge through a multiple of 10. 8 + 7 = 15  **48 + 37** = 85  **Alternatives (for some children)** 48 + 37 = 85 	This method will be a jotting approach, and may look like the following examples: - **48 + 37** 48 + 30 = 78 78 + 7 = 85 Or 48 + 30 + 7 = 85 

Year group	Main method	Alternative method(s)
	Stage 2: Partitioning	**Partition one of the numbers**
Year 2 / 3 Add speech bubbles	Record steps in addition using partitioning: Initially as a jotting: - **58 + 87** = 50 + 80 + 8 + 7 = 130 + 15 = 145 Or 50 + 80 = 130 8 + 7 = 15 130 + 15 = 145 Partitioned numbers are then written under one another: - 50 8 80 7 130 15 = 145	**58 + 87** ***This method is basically a 'jotting' version of the number line method*** Or 87 + 50 = 137 58 + 80 = 138 137 + 8 = 145 138 + 7 = 145 Or 87 + 50 + 8 = 145 One popular jotting approach is: - 5 8 + 8 7 130 + 15 = 145

Stage 3: Expanded method in columns	
Year 2 (Simple examples to introduce the expanded method to the children. Many children would continue to answer these calculations mentally or using a simple jotting – See **Stage 2**)	**A. Single 'carry' in units** **B. 'Carry' in units and tens** Adding the units first: 67 + 24: 11, 80, 91 58 + 87: 15, 130, 145 ***Adding the units first gives the same answer as adding the tens first*** Refine over time to adding the units digits first consistently, with harder calculations **457 + 76** **538 + 286** **457** **+ 76**: 13, 120, 400, 533 **Then** **538** + 286: 14, 110, 700, 824 The time spent practising expanded method will depend on security of number facts recall and understanding of place value.
Stage 4: Column method	
Year 3 onwards ***Record carry digits below the line*** **Years 5-6**	**58 + 87** **457 + 76** **538 + 286** 58 + 87 = 123 (carry 1 1) Then 457 + 76 = 533 (carry 1 1) Then 538 + 286 = 824 (carry 1 1) ***Use the words 'carry ten' and 'carry hundred', not 'carry one'*** Once confident, use with larger whole numbers and decimals. Return to expanded if children make repeated errors **2467 + 785** **4824 + 2369** **46.73 + 78.6** 2467 + 785 = 3252 (carry 1 1 1) 4824 + 2369 = 7193 (carry 1 1) 46.73 + 78.60 = 125.33 (carry 1 1 1)

Written methods for subtraction of whole numbers

The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence.

To subtract successfully, children need to be able to:

- **recall all addition and subtraction facts to 20;**
- **subtract multiples of 10 (such as 160 – 70) using the related subtraction fact,16 – 7, and their knowledge of place value;**
- **partition two-digit and three-digit numbers into multiples of one hundred, ten and one in different ways (e.g. partition 74 into 70 + 4 or 60 + 14).**

Note: It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for subtraction.

Children need to acquire **one efficient written method of calculation for** subtraction which they know they can rely on **when mental methods are not appropriate.**

But, they should look at the actual numbers each time they see a calculation and decide whether or not their favoured method is most appropriate (e.g. If there are zeroes in a calculation such as 2006 -128 then the 'counting on' approach may well be the best method in that particular instance

Therefore, when subtracting, whether mental or written, children will mainly choose between two main strategies: -

Taking away (Counting Back) ***Complementary Addition (Counting On)***

When should we count back and when should we count on?

This will alter depending on the calculation (see below), but often the following rules apply

If the numbers are far apart, or there isn't much to subtract (278 – 24) then count back.

If the numbers are close together (206 – 188), then count up

In many cases, either strategy would be suitable

Year group	Subtraction by counting back (or taking away)	Subtraction by counting up (or complementary addition)
R	Counting back from a given number, “one less” and number rhymes. Practical tasks relating to taking away. Using number lines or number tracks and fingers / apparatus to calculate subtraction.	
Year 1	Using number lines or number tracks and fingers / apparatus to calculate subtraction. Count on to find the difference.	

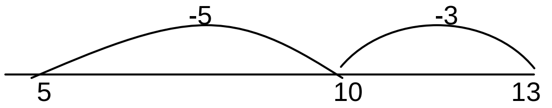
Stage 1: Using the empty number line

The empty number line helps to record or explain the steps in mental subtraction.
It is an ideal model for **counting back** and **bridging ten**, as the steps can be shown clearly.
It can also show **counting up** from the smaller to the larger number to **find the difference,**

Year 2

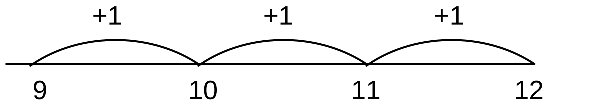
The steps often bridge through a multiple of 10.

13 – 8 = 5



Small differences can be found by counting up

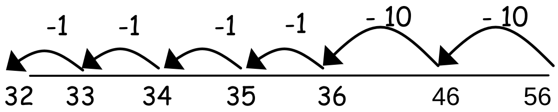
12 – 9 = 3



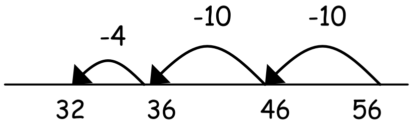
Year 2/3

For 2 digit numbers, count back in 10s and 1s

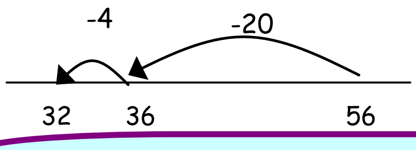
56 – 24 = 32



Then subtract the units in a single jump



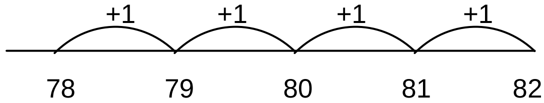
Then subtract tens and units in single jumps



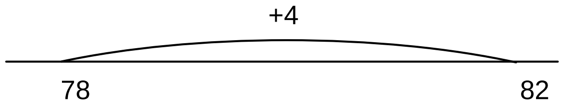
For 2 (or 3) digit numbers close together, count up

82 – 78 = 4

First, count in ones



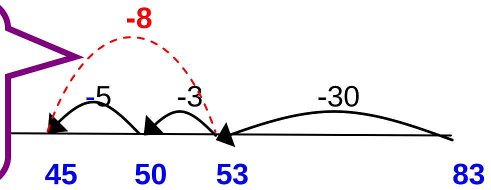
Then, use number facts to count in a single jump



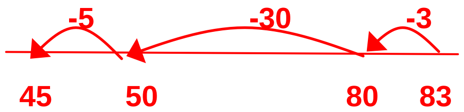
Some numbers (83 – 38) can be subtracted just as quickly either way.

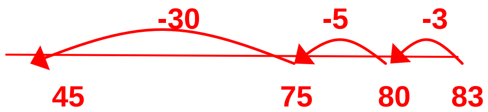
83 – 38 = 45

Partition 38. Take away 30 then take away 8 (-3 -5)

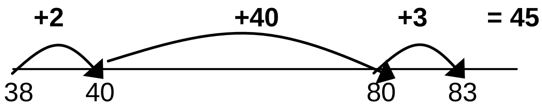


Alternatives





Count up from the smaller to the larger number.



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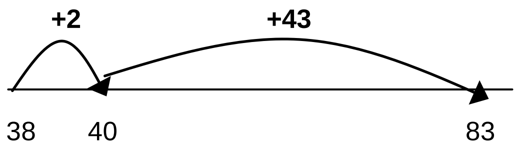


	Stage 2: Subtraction by counting back Expanded method	Subtraction by counting up Number lines (continued)

Year 2

Introduce the expanded method with 2 digit numbers to explain the process.
Partition both numbers into tens and units.
Exchange from the tens to the units.

83 – 38

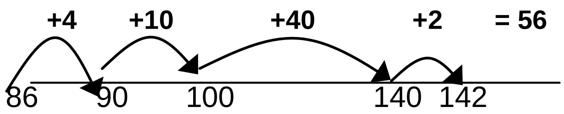
```
    80   3        70  13          70    1
-   30   8        30   8          80̸    3
                               -  30    8
                                  40    5
```

Exchange from hundreds to tens and tens to units

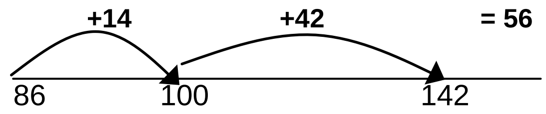
142 - 86

```
100  40  2     100 30 12     130    1
-    80  6         80  6     1̸00  4̸0  2
                                  80  6
                                  50  6
```

142 – 86



Or (in fewer steps)



Year 2

Take the method into three digit numbers
Subtract the units then the tens then the hundreds

A

Demonstrate without exchanging first

784 - 351

```
   700   80   4
-  300   50   1
   400   30   3
```

For examples without exchanging, the number line method takes considerably longer than mental partitioning or expanded.

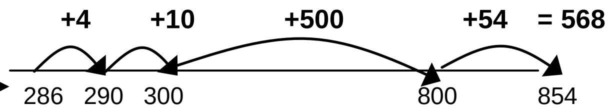
B

Move towards exchanging from hundreds to tens and tens to ones

854 - 286

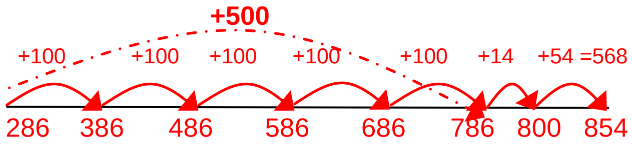
```
   800   50   4          700   140    1
-  200   80   6          8̸00   5̸0    4
                      -  200   80    6
                         500   60    8
```

854 - 286



Or (the efficient method)

Alternative (count the hundreds first)



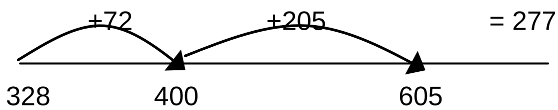
C

Use some examples which include the use of zeros

605 – 328

```
                          90
   600    0   5     500   1̸00    1
-  300   20   8     6̸00   0̸     5
                 -  300   20    8
                    200   70    7
```

For numbers containing zeros, counting up is often the most reliable



Continue to use expanded subtraction until both number facts and place value are considered to be very secure

Stage 3: Standard method (decomposition)

Mainly Y3 onwards

Decomposition relies on secure understanding of the expanded method, and simply displays the same numbers in a contracted form.

(Using example B from Stage 2)

854 – 286

```
 7 14  1
 8  5  4
-2  8  6
 5  6  8
```

Continue to refer to digits by their actual value, not their digit value, when explaining a calculation. E.g. One hundred and forty subtract eighty.

Again, use examples containing zeros, remembering that it may be easier to count on with these numbers (see Stage 2)

(Using example C from Stage 2)

605 – 328

```
   9
 5 1  1
 6 0  5
-3 2  8
 2 7  7
```

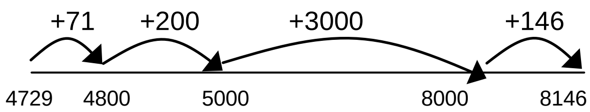
The counting up method is often used in Years 5 and 6 for children whose mental recall is weak, or who require a visual image to support their thinking.

Move onto examples using 4 digit (or larger) numbers and then onto decimal calculations.

814 6 – 472 9

```
 7 1 3 1
 8 1 4 6
-4 7 2 9
 3 4 1 7
```

814 6 – 472 9



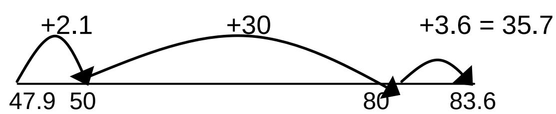

```
=  3000
    146
    200
     71
  341 7
```

Both methods can be used with decimals, although the counting up method becomes less efficient and reliable when calculating with more than two decimal places.

83.6 – 47.9

```
 7 12 1
 8  3. 6
-4  7. 9
 3  5. 7
```

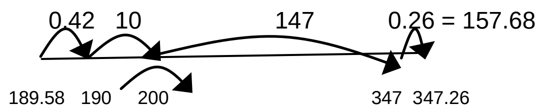
83.6 – 47.9



347.26 – 189.58

```
  1 13 16 11 1
  3  4  7 . 2 6
- 1  8  9 . 5 8
  1  5  7 . 6 8
```

347.26 – 189.58



Written methods for multiplication of whole numbers

The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence.

These notes show the stages in building up to using an efficient method for two-digit by one-digit multiplication by the end of Year 4, two-digit by two-digit multiplication by the end of Year 5, and three-digit by two-digit multiplication by the end of Year 6.

To multiply successfully, children need to be able to:

- **recall all multiplication facts to 10 × 10;**
- **partition number into multiples of one hundred, ten and unit;**
- **work out products such as 70 × 5, 70 × 50, 700 × 5 or 700 × 50 using the related fact 7 × 5 and their knowledge of place value;**
- **add two or more single-digit numbers mentally;**
- **add multiples of 10 (such as 60 + 70) or of 100 (such as 600 + 700) using the related addition fact, 6 + 7, and their knowledge of place value;**
- **add combinations of whole numbers using the column method (see above).**

Note:

Children need to acquire **one efficient written method of calculation for** multiplication which they know they can rely on **when mental methods are not appropriate.**

It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for multiplication.

These mental methods are often more efficient than written methods when multiplying.

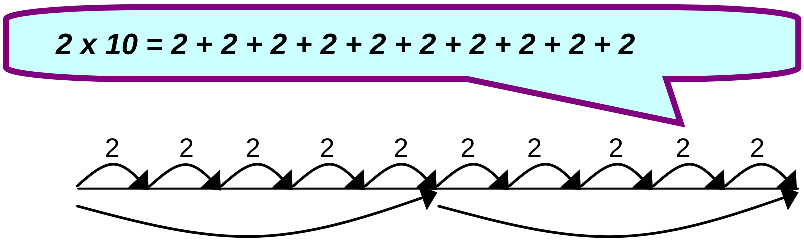
Use partitioning and grid methods until number facts and place value are secure

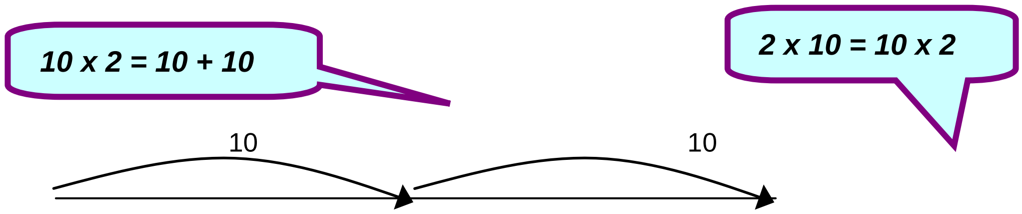
For a calculation such as 25 x 24, a quicker method would be 'there are four 25s in 100 so 25 x 24 = 100 x 6 = 600

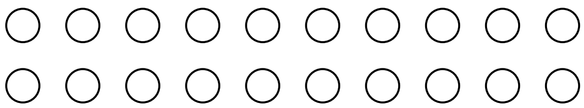
When multiplying a 2 digit x 3 digit number (or a 3digit x 3 digit number), the standard method is usually the most efficient

At all stages, use known facts to find other facts. E.g. Find 7 x 8 by using 5 x 8 (40) and 2 x 8 (16)

<table>
<tr><th></th><th>Expanded multiplication</th><th>Standard ‘compact’ multiplication</th></tr>
<tr><td>R</td><td colspan="2">Repeated addition, counting in 10s, and 2s.</td></tr>
<tr><td></td><td colspan="2"></td></tr>
<tr><th>Year group</th><th colspan="2">Stage 1: Number lines and mental methods</th></tr>
<tr><td>Year 1 and 2</td><td colspan="2">Repeated addition, counting in 10s, 5s and 2s. Counting larger numbers by grouping into 5s and 10s
Begin by building on the understanding that multiplication is repeated addition, using arrays and number line s to support the thinking.

Using a number line


Or


Using an array

10 x 2 = 20
2 x 10 = 20</td></tr>
</table>

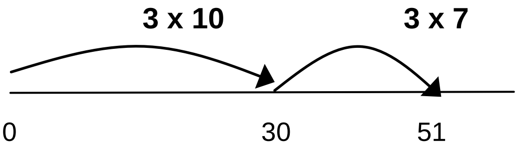
Year 3

Extend the above methods to include the 3, 4, 8 times tables then begin to partition using **jottings and number lines.**

3 x 17

x 3 | **10** + **7** → 30 + 21 = **51**

becomes



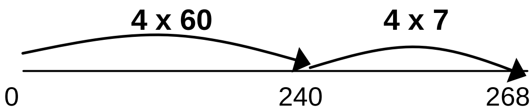
Or

10 x 3 = 30
7 x 3 = 21
51

Extend the methods above to calculations which give products greater than 100.

4 x 67

x4 | 60 + 7 **x 4** → 240 + 28 = **268**



Or

60 x 4 = 240
7 x 4 = 28
268

Each of these methods can be used in the future if children find expanded or standard methods difficult

Extend to using these methods with all tables to 10 x 10.

Year group	Stage 2: Written methods – Short multiplication	
	Grid multiplication	**Vertical multiplication (Expanded method into standard)**

Later Year 3 onwards

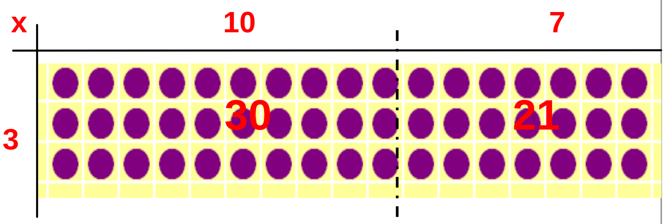
Grid multiplication

The grid method of multiplication is a simple, alternative way of recording the jottings shown previously.

3 x 17

	10	7	
3	**30**	**21**	**= 51**

If necessary (for some children) it can initially be taught using an array to show the actual product.



4 x 67

	60	7	
4	**240**	**28**	**= 268**

Use all tables with more complex calculations

7 x 89

	80	9	
7	**560**	**63**	**= 623**

Move onto HTU x U

4 x 378

	300	70	8
4	**1200**	**280**	**32**

= 1512

The grid method may continue to be the main method used by children whose mental and written calculation skills are weak, or children who need the visual link to place value.

Vertical multiplication (Expanded method into standard)

The expanded method links the grid method to the standard method.

It still relies on partitioning the tens and units, but sets out the products vertically.

Children will use the expanded method until they can securely use and explain the standard method.

When setting out calculations vertically, begin with the units first (as with addition and subtraction)

4 x 67

```
  6 7
x   4
2 6 8
  2
```

Place the 'carry' digit below the line

7 x 89

```
  8 9
x   7
 6 2 3
  6
```

Where numbers are difficult to add mentally, try to use the expanded or standard methods

4 x 378

```
  3 7 8
x     4
1 5 1 2
 3  3
```

In all calculations, refer to the actual value of the digits.
E.g. 4 multiplied by by 70, not 7

Stage 4: Long multiplication: TU xT U

Year group	Short multiplication	Vertical 'standard' long multiplicaton

Years 5 & 6

Short multiplication

Extend the grid method to TU × TU, asking children to estimate first. '

38 x 57

38 × 57 is approximately 40 × 60 = 2400.

x	50	7	
30	1500	210	1 7 1 0
8	400	56	4 5 6
			2 1 6 6

1

Add the two products in each row

Add these sums for the overall product

The grid method is often the 'choice' of many children in Years 5 and 6, and is the method that they will mainly use for long multiplication.

Vertical 'standard' long multiplicaton

Children should only use the 'standard' method of long multiplication if they can regularly get the correct answer using this method.

38 x 57

38 × 57 is approximately 40 × 60 = 2400.

```
    3 8
  x 5 7
  2 6 6
   2 5
1 9 0 0
 1  4
2 1 6 6
```

Stage 4: Long multiplication: HTU x TU

Year 5 and 6

Short multiplication

For HTU x TU, grid method is quite inefficient, and has much scope for error due to the number of 'part-products' that need to be added.

Use this method when you find the standard method to be unreliable or difficult.

423 x 68

423 × 68 is approximately 400 × 70 = 28000.

X	60	8	
400	24000	3200	27200
20	1200	160	1360
3	180	24	204
			28764

Vertical 'standard' long multiplicaton

Many children choose the standard method. For HTU x TU calculations It especially efficient, and less prone to errors when mastered.

423 x 68

423 × 68 is approximately 400 × 70 = 28000.

```
    4 2 3              4 2 3
     x 6 8    or   x    6 8
                       1 2
  3 3 8 4          3 3 8 4
     1 2             1 4
2 5 3 8 0        2 5 3 8 0
  1 1
2 8 7 6 4        2 8 7 6 4
   1                1
```

As before, either carry the 'carry'digits mentally or decide on your own favoured position for them.

Written methods for division of whole numbers

The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence.

These notes show the stages in building up to long division through Years 4 to 6 – first long division TU ÷ U, extending to HTU ÷ U, then HTU ÷ TU, and then short division HTU ÷ U.

To divide successfully in their heads, children need to be able to:

- **understand and use the vocabulary of division – for example in 18 ÷ 3 = 6, the 18 is the dividend, the 3 is the divisor and the 6 is the quotient;**
- **partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways;**
- **recall multiplication and division facts to 10 × 10, recognise multiples of one-digit numbers and divide multiples of 10 or 100 by a single-digit number using their knowledge of division facts and place value;**
- **know how to find a remainder working mentally – for example, find the remainder when 48 is divided by 5;**
- **understand and use multiplication and division as inverse operations.**

Children need to acquire **one efficient written method of calculation for** subtraction which they know they can rely on **when mental methods are not appropriate.**

Note: It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for division.

To carry out expanded and standard written methods of division successful, children also need to be able to:

- **understand division as repeated subtraction;**
- **estimate how many times one number divides into another – for example, how many sixes there are in 47, or how many 23s there are in 92;**
- **multiply a two-digit number by a single-digit number mentally;**
- **understand and use the relationship between single digit multiplication, and multiplying by a multiple of 10. (e.g. 4 x 7 = 28 so 4 x 70 = 280 or 40 x 7 = 280 or 4 x 700 = 2800.)**
- **subtract numbers using the column method.**

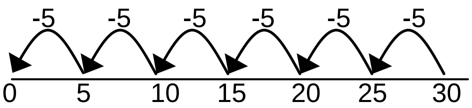
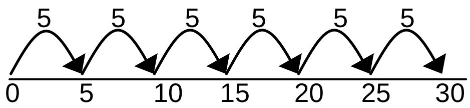
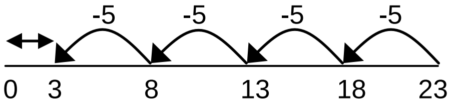
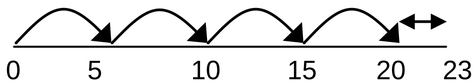
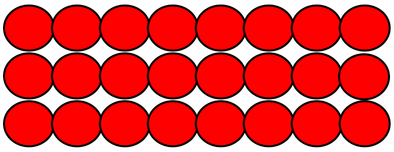
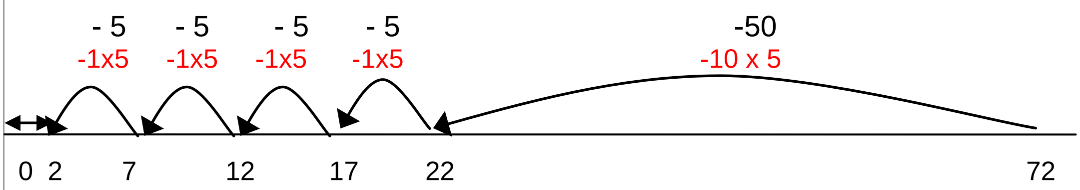
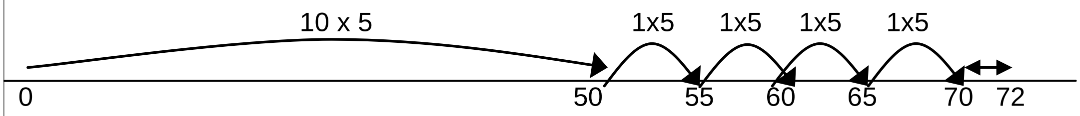
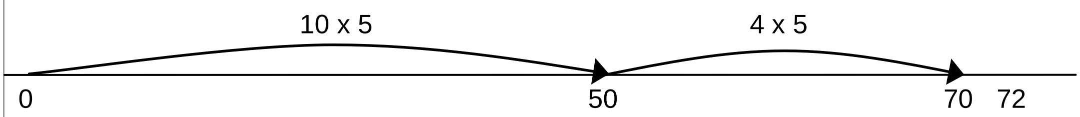
The above points are crucial. If children do not have a secure understanding of these prior learning objectives then they are unlikely to divide with confidence or success, especially when attempting the 'chunking' method of division.

For example, without a clear understanding that 72 can be partitioned into 60 and 12, 40 and 32 or 30 and 42 (as well as 70 and 2), it would be difficult to divide 72 by 6, 4 or 3 using the 'chunking' method.

72 ÷ 6 'chunks' into 60 and 12

72 ÷ 4 'chunks' into 40 and 32

72 ÷ 3 'chunks' into 30 and 42 (or 30, 30 and 12)

Stage 1: Number line division and mental division (pre chunking)	
Year group	
Yr 1	Calculate using pictorial representation and arrays.
Year 2	***Start to emphasise grouping over sharing as a more efficient way to divide.*** Beginning with the number line is an excellent way of linking division to multiplication. It can show division both as repeated subtraction, and as counting forward to find how many times one number 'goes into' another. **30 ÷ 5** or  It also helps the children to deal with remainders. **23 ÷ 5 = 4 r 3** or  Some children will continue to use arrays to develop their understanding of division, and to link to multiplication facts.  This array can show 24 ÷ 3 and 24 ÷ 8
Years 3 and 4	The number line is also an excellent way of introducing the 'chunking' approach. **72 ÷ 5 = 14 r 2**  Or  Into a more efficient 

Regularly stress the link between multiplication and division, and how children can use their tables facts to divide by counting forwards in steps.

In lower KS2, children need a great deal of practice in mentally 'chunking' to develop their understanding of division. They can use an informal jotting to support their thinking.

These mental methods for dividing TU ÷ U are usually based on partitioning in different ways.

72 ÷ 5	**72 ÷ 6**	**72 ÷ 4**	**72 ÷ 3**
72 ÷ 5 = 14 r 2	72 ÷ 6 = 12	72 ÷ 4 = 13	72 ÷ 3 = 24
50 20 2	60 12	40 32	60 12
10x5 4x5	10x6 2x6	10x4 8x4	20x3 4x3

Stage 2: Short division 'chunking'

Year group	Chunking – TU ÷ U
Year 3	• 'Short' division of **TU ÷ U** introduces the 'chunking' method. • This becomes more useful with **HTU ÷ U** and later for long division. • Chunking helps to consolidate the link between division and repeated subtraction. •

Once children can understand chunking for TU ÷ U, they move on to HTU ÷ U quite quickly.

When chunking we repeatedly subtract multiples or 'chunks' of the divisor.

51 ÷ 3 = 17

A 'chunk' of 10 lots of the divisor is the most common

```
   51
 - 30   10 x 3)
   21
 - 21    7 x 3)

        17
```

Introduce chunking using simple examples that only require a single chunk of 10 lots of the divisor.

Progress to examples which may require more than one chunk of 10 lots of the divisor

87 ÷ 3 = 29

```
   87
 - 30   10 x 3
   57
 - 30   10 x 3
   27
 - 15    5 x 3
   12
 - 12    4 x 3
        29
```

Begin by subtracting several chunks , but then try to find the biggest chunks of the divisor that can be subtracted.

OR

```
   87
 - 60   20 x 3
   27
   27    9 x 3
        29
```

Chunking - HTU ÷ U

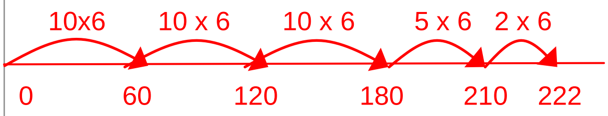
Year group	'Chunking' examples	Number line alternatives
Year 4	Progress quickly to HTU ÷ U examples. Again, some children will initially subtract many chunks of the divisor.	Remember, the number line method can still be used for children who find the subtraction 'chunking' difficult. They are still finding chunks of the divisor but are counting on rather than counting back.

222 ÷ 6 = 37

```
   222
-   60    10 x 6
   162
-   60    10 x 6
   102
-   60    10 x 6
    42
-   30     5 x 6
    12
    12     2 x 6

          37
```

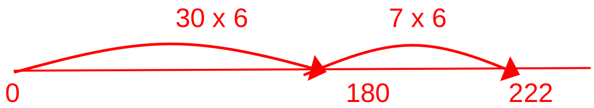
222 ÷ 6 = 37



These are inefficient. Try to find the largest possible chunks of the divisor to shorten the calculation.

```
   222
-  180    30 x 6
    42
    42     7 x 6
          37
```

If children have secure recall of x and ÷ facts, their chunking will soon become efficient.



An estimate at the start will help children to find the largest chunks. If 6 x 3 = 18and 6 x 4 = 24 then 6 x 30 = 180 and 6 x 40 = 240. Therefore the answer will be between 30 and 40

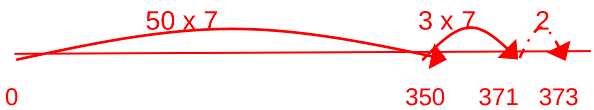
Make sure that you include examples that use remainders.

373 ÷ 7 = 53 r 2

```
   373
-  350     50 x 7
    23
    21      3 x 7
     2     53
```

Answer = **53 r 2**

373 ÷ 7 = 53 r 2



Stage 3: Short division – the compact method

Late Year 4/ Year 5 / Year 6

Only use this 'standard' method when children have had lots of experience with the chunking method and are confident with all

The compact or 'bus shelter' method of short division is often introduced far too early (in Years 3 and 4). Although children who can recall their division facts are able to get the correct answer using this method, they have little understanding of why it works, and are not using the place value of each digit.
By leaving this method until Year 6, children can develop greater confidence in their actual understanding of division, and will hopefully be able to then apply the 'chunking' method to long division, as they will have had much more practice
This compact method can then be introduced to improve their speed in short division.

Initially, introduce this method by linking it to 'chunking'.

87 ÷ 3 = 29

```
    20 + 9
3 )60 +27
```

Then, refine the method into the traditional format, ensuring that all initial teaching is accompanied by a clear explanation of how this method works (see speech bubbles)

```
   2
3 ) 8 ²7
```

From 80, what is the largest number of 10s that will divide exactly by 3?
60 (or 6 tens) ÷ 3 = 20 (or 2 tens). Carry the remaining 20 to the units.

```
   2 9
3 ) 8 ²7
```

What is 27 divided by 3

When this method is secure for TU ÷ U then quickly progress to HTU ÷ U
Again, begin by briefly linking the method to 'chunking', using numbers where there is no carrying in the hundreds.

222 ÷ 6 = 37

```
    30 +  7
6 )180 + 42
```

Refine the method, whilst clearly explaining the place value understanding.

```
     3
6 ) 2 2 ⁴2
```

From 220,what is the largest number of 10s that will divide exactly by 6?
220 ÷ 6 = 30 (or 3 tens). Carry the remaining 40 to the units.

```
     3 7
6 ) 2 2 ⁴2
```

What is 42 divided by 6?

An alternative is to say ***'How many 6s in 220 – the answer must be a multiple of 10'***

Finally, introduce examples of HTU ÷ U where there are also hundreds that need carrying, and where there are remainders.

583 ÷ 4 = 145 r 3

```
       100 +  40 +  5     R 3
   4 ) 400 + 160 + 23
```

Continue to emphasise the place value until the children are secure with this method.

```
         1
   4 ) 5 ¹8  3
```

From 500,what is the largest number of 100s that will divide exactly by 4?

400 ÷ 4 = 100. Carry the remaining 100 to the ten.

Or, 'How many 4s in 500? The answer must be a multiple of 100.

```
         1   4
   4 ) 5  ¹8  ²3
```

From 180,what is the largest number of 10s that will divide exactly by 4?

180 ÷ 4 = 40. Carry the remaining 20 to the units.

Or, 'How many 4s in 180? The answer must be a multiple of 10.

```
         1   4   5  R3
   4 ) 5  ¹8  ²3
```

What is 23 divided by 4?

Stage 4: Long division - chunking

Year 6 (more able)

More able children can now tackle long division, beginning with HTU ÷ TU and then moving onto ThHTU ÷ TU. They will use the chunking method.

967 ÷ 26 = 37 R5

```
     967
  -  520   20  x  26
     447
  -  260   10  x  26
     187
  -  130    5  x  26
      57
  -   52    2  x  26
       5             =  37 R 5
```

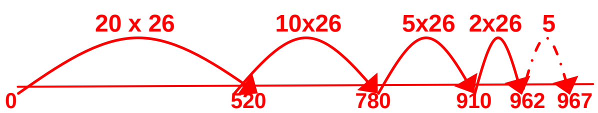
Try to find the largest chunks if possible.

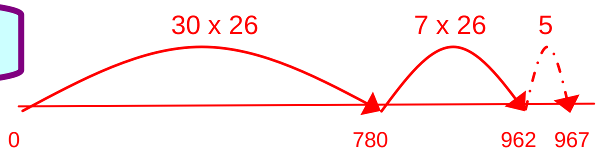
```
     967
  -  780   30  x  26
     187
  -  182    7  x  26
       5             =  37 R 5
```

This refined answer is actually the traditional long division method, but with place value kept secure, and with the advantage of being able to choose smaller chunks when necessary

As before, children can choose to use the number line method if they find forward chunking easier.

967 ÷ 26 = 37 R5





At this stage more able children can also create their own 'mental chunking'

260, 520, 780, 910, 962 =
10 10 10 5 2 37 R 5

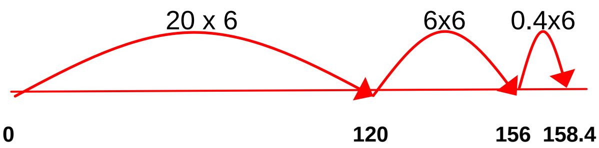
Stage 5: Division of decimals

For some children, again at Level 5, the chunking method can be used for division of decimal numbers.

158.4 ÷ 6 = 26.4

	158.4	
-	120.0	20 x 6
	38.4	
-	36.0	6 x 6
	2.4	
-	2.4	0.4 x 6
		= **26.4**

158.4 ÷ 6 = 26.4



Extend into division by decimals.

22.4 ÷ 0.8 = 28

22.4	
16.0	20 x 0.8
6.4	
6.4	8 x 0.8

22.4 ÷ 0.8 = 28